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TECHNICAL NOTES

Radiative transfer through a medium of silica fibres oriented in parallel planes

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INTRODUCTION

THE STUDY of heat transfer through porous materials is an active research sector, with many applications in thermal insulation. Particular attention has been focused on radiative transfer because it represents 40% of all heat transfer at room temperature for the materials used in building insulation, for example.

The purpose of this study is to calculate the pure radiative transfer for a layer of silica fibers. In many previous studies (see Ozisik for a general review [1]), assumptions such as grey behaviour or isotropic scattering were largely made to simplify the analytic solution of radiative transfer. Therefore, we will focus on two aspects: the non-grey behaviour of the material [2, 3] and the influence of its morphology (anisotropic orientation and diameter dispersion of fibres composing the medium).

Lee [4, 5] showed the influence of fibre distribution and particularly of their orientation, by determining especially the radiative properties of media with fibres oriented in planes parallel to the boundaries.

We adapted this formulation to a real medium composed of silica fibres in order to determine its properties as functions of wavelength. In addition transmission measurements allowed us to verify our numerical results.

ANALYTICAL FORMULATION

Let us consider radiation with wavelength λ and specific intensity $L_\lambda(\Delta,s)$ where λ refers to the wavelength, Δ to the direction and s to the position in space. By interaction with a fibre element of length $\mathrm{d}s$, the radiation is modified by effects contributing to gains or losses of intensity. The equation of radiative transfer for an absorbing, scattering and emitting medium is

$$\frac{\mathrm{d}L_\lambda(\Delta,s)}{\mathrm{d}s} = \sigma_{a\lambda}L_{o\lambda}(T)-(\sigma_{a\lambda}+\sigma_{s\lambda})L_\lambda(\Delta,s) + \frac{1}{4\pi}\int_{\Omega'=4\pi}\sigma_{s\lambda}P_\lambda(\Delta'\rightarrow\Delta)L_\lambda(\Delta',s)\,\mathrm{d}\Omega'. \quad (1)$$

Consider a medium of specific symmetry: a medium with fibres stratified in planes parallel to the boundaries, but with random azimuthal orientation (Figs. 1 and 2). The equation of radiative transfer then becomes for a one-dimensional transfer

$$\mu\frac{\mathrm{d}L_\lambda(y,\omega,\mu)}{\mathrm{d}y} = \sigma_{a\lambda}L_{o\lambda}(T)-(\sigma_{a\lambda}+\sigma_{s\lambda})L_\lambda(y,\omega,\mu) + \frac{1}{4\pi}\int_{-1}^{1}\int_0^{2\pi}\sigma_{s\lambda}P_\lambda(\Delta'\rightarrow\Delta)L_\lambda(y,\mu',\omega')\,\mathrm{d}\omega'\,\mathrm{d}\mu' \quad (2)$$

where $\mu = \cos\xi$.

The problem of radiative transfer through this material therefore requires knowledge of the monochromatic coefficients previously defined: $\sigma_{a\lambda}$, $\sigma_{s\lambda}$ and $P_\lambda(\Delta'\rightarrow\Delta)$.

We used Kerker's [6] and Lind and Greenberg's [7] results to calculate the scattering by a silica fibre. Lee's theory allows the determination of fibrous media radiative properties as functions of wavelength and material properties (fibre size and complex refractive index of the medium). For this calculation, we consider fibres oriented in the (x, z) plane, such that $\xi_f = \pi/2$ (Fig. 2).

Absorption, scattering and extinction coefficient calculation

By definition, the extinction monochromatic coefficient $\sigma_{e\lambda}$ is

$$\sigma_{e\lambda} = \sigma_{a\lambda}+\sigma_{s\lambda}. \quad (3)$$

It highlights the total intensity loss in the medium.

Coefficients $\sigma_{i\lambda}$ are defined as functions of efficiencies $Q_{i\lambda}$ as [5]

$$\sigma_{i\lambda}(\xi) = \frac{rN}{\pi}\int_0^{2\pi}Q_{i\lambda}(\phi)\delta\left(\xi_f-\frac{\pi}{2}\right)\mathrm{d}\omega_f \quad (i = \mathrm{e, a, s}) \quad (4)$$

with

$$\sin\phi = \sin\xi\cos(\omega-\omega_f) \quad (5)$$

$$N = \frac{f_v}{\pi r^2} \quad (6)$$

where f_v is the volume fraction of fibres in the medium.

The average coefficient is also defined as

$$\overline{\sigma_{i\lambda}} = \frac{1}{2\pi}\int_\Omega\sigma_{i\lambda}(\xi)\,\mathrm{d}\Omega = \int_0^{\pi/2}\sigma_{i\lambda}(\xi)\sin\xi\,\mathrm{d}\xi. \quad (7)$$

Backscatter formulation

Instead of calculating the phase function $P_\lambda(\Delta'\rightarrow\Delta)$, we established a parameter usually found in radiative transfer studies using the two flux model: the backscatter factor B_λ such as

$$B_\lambda(\xi) = \frac{1}{4\pi}\int_\sigma(\sigma_{s\lambda}\cdot P_\lambda)\,\mathrm{d}\Omega. \quad (8)$$

Using Lee's notation [5]

$$B_\lambda(\xi) = \frac{N\lambda}{2\pi^3}\int_0^{2\pi}\int_{\theta_c}^{\theta_c+\pi}i(\theta,\phi)\delta\left(\xi_f-\frac{\pi}{2}\right)\mathrm{d}\theta\,\mathrm{d}\omega_f \quad (9)$$

where θ_c is the critical angle of observation defined with regard to the stratification plane as

$$\cos\theta_c = \frac{\sin\xi\sin(\omega-\omega_f)}{\sqrt{(1-\sin^2\xi\cos^2(\omega-\omega_f))}} \quad (9')$$

NOMENCLATURE

B_λ	backscatter factor
f_v	volume fraction of fibres
L	thickness of the medium
L_λ	intensity of the radiation
$L_{o\lambda}$	black-body intensity
N	number density of fibres
P_λ	phase function
q_λ	radiative flux
r	radius of fibre
s	position in space
T	temperature
y	thickness along the medium.

Greek symbols

Δ	direction
ε	emissivity
θ	angle of observation
θ_c	critical angle of observation
λ	wavelength
λ_r	radiative conductivity
μ	$\cos\xi$
ξ	polar angle
ϕ	angle of incidence
$\sigma_{a\lambda}$	absorption coefficient
$\sigma_{e\lambda}$	extinction coefficient
$\sigma_{s\lambda}$	scattering coefficient
ω	azimuthal angle
Ω	solid angle.

Symbols

⌓	upper hemisphere
◡	lower hemisphere.

Subscripts

a	absorption
e	extinction
f	fibres
s	scattering.

and where $i(\theta, \phi)$ is the relative intensity of the wave scattered by a fibre, as defined by Kerker [6].

We can calculate an average monochromatic backscatter factor divided by the extinction coefficient as

$$\overline{B_\lambda} = \frac{1}{2\pi\overline{\sigma_{e\lambda}}}\int_{\cap} B_y(\xi)\,\mathrm{d}\Omega. \tag{10}$$

Notion of radiative conductivity

Using a two flux model with the hypothesis of hemispherical isotropy of intensity of Schuster and Schwarzchild and with the boundary conditions (ε_1, T_1) and (ε_2, T_2), we can define an effective pure radiative flux q_λ such that [5]

$$q_\lambda = \frac{e_1 - e_2}{\left(\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1\right)+(\overline{\beta_{a\lambda}}+2\overline{B_\lambda})\overline{\sigma_{e\lambda}}L} \tag{11}$$

with e_1 and e_2 designating the boundary emissive power. L is the medium thickness and $\beta_{a\lambda}$ the average monochromatic absorption ratio so that

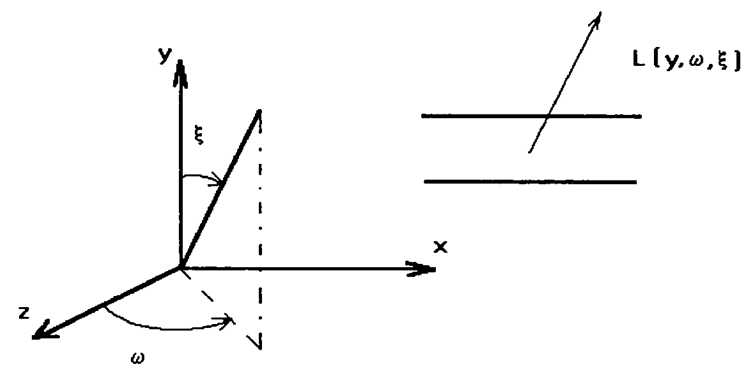

FIG. 1. Orientation in space.

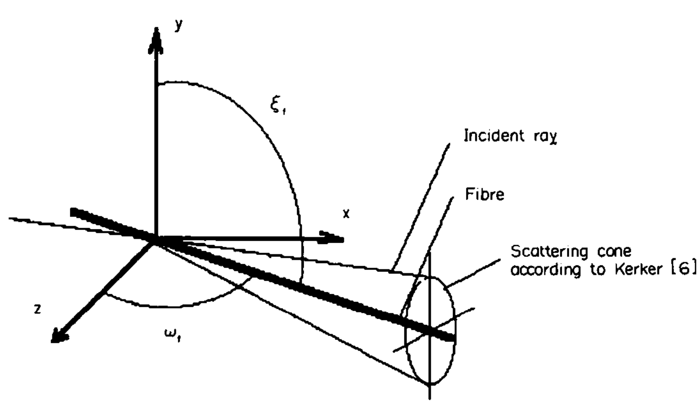

FIG. 2. Geometry of scattering for a fibre in the (x, z) plane according to Lee [5].

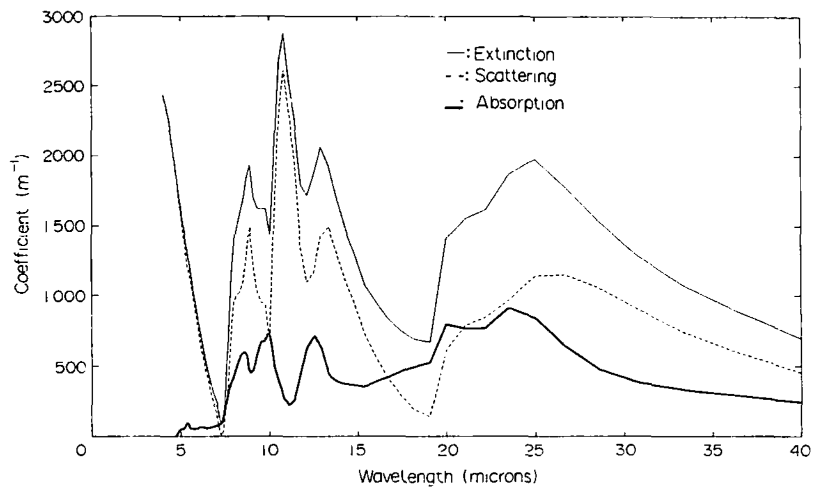

FIG. 3. Variation of the extinction, the scattering and the absorption coefficient with the wavelength.

$$\overline{\beta_{a\lambda}} = \frac{\overline{\sigma_{a\lambda}}}{\overline{\sigma_{e\lambda}}}. \tag{12}$$

The radiative conductivity is also defined as

$$\lambda_r = \int_0^\infty q_\lambda \, d\lambda x \frac{L}{T_1 - T_2}. \tag{13}$$

Expression (11) and the radiative conductivity definition according to Tong and Tien [8, 9] allow us to write a relation which takes into account the non-grey behaviour of the medium

$$\lambda_r = \frac{\displaystyle\int_0^\infty \frac{L_{o\lambda}(T)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + \beta_\lambda L} d\lambda}{\displaystyle\int_0^\infty L_{o\lambda}(T) \, d\lambda} \times 4\sigma T^3 L \tag{14}$$

where

$$\beta_\lambda = (\overline{\beta_{a\lambda}} + 2\overline{B_\lambda})\overline{\sigma_{e\lambda}}$$

σ represents Planck's constant and $L_{o\lambda}(T)$ the black-body emissive power at temperature $T = (T_1 + T_2)/2$.

For the calculation, we expressed $L_{o\lambda}$ using Planck's law and we limited the integration to the domain (4 μm, 40 μm) which corresponds to the most part of the black-body emission at room temperature.

RESULTS

We calculated the radiative properties of a medium of silica fibres having the following properties:

- volume mass of the silica composing the medium: 2500 kg m^{-3};
- volumic mass of the fibrous layer: 10 kg m^{-3};
- diameter of fibres between 0.5 and 10 μm;
- fibres oriented in stratification planes parallel to the boundaries.

Figures 3–5 present the variations of the radiative properties variations as functions of wavelength for fibres with a diameter of 6 μm. On these curves, singularities occur at 7.3 and 19.3 μm associated with the Christiansen effect. Indeed, at these wavelengths, the refractive index n of the two phases composing the medium (air and silica) are identical, the fibrous medium behaves as a homogeneous body and the incident wave is not scattered.

The intensity loss is then essentially due to absorption, relatively weak in this case.

From the results obtained for the medium radiative properties, we calculated the radiative conductivity, from relation (14), considering $\varepsilon_1 = \varepsilon_2 = 1$.

Figure 6 confirms the existence of a thickness effect. Indeed, the conductivity increases with the medium thickness to reach the optically thick medium limit (10.75 mW m^{-1} K^{-1} for a fibre diameter of 6 μm).

The influence of the medium morphology is shown on Fig. 7. With the hypothesis of an optically thick medium the variations of λ_r with fibre diameter are plotted. It appears that a diameter of 2 μm minimizes the heat transfer. Using the fibers diameter distribution histogram presented in Fig. 8 it is possible to calculate the effective radiative conductivity of our medium: $\lambda_r = 10.8$ mW m^{-1} K^{-1}. This theoretical value differs from some experimental results obtained from measurements using a guarded hot plate system. (For example from ref. [2]: $\lambda_r = 20$ mW m^{-1} K^{-1}.) However, there is a progress in comparison with previous calculations on a two-flux model considering a fibre isotropic distribution: $\lambda_r = 8.4$ mW m^{-1} K^{-1} [2].

The difference between theoretical and experimental values probably comes from the assumptions we set when we used the two flux model.

EXPERIMENTAL VALIDATION

Figure 9 presents the aspect of the directional normal spectral transmittance between 4 and 40 μm for a slab with a thickness of 5 mm. The experimental curve has been obtained with a Perkin Elmer 357 spectrometer on a medium with characteristics identical to those used in our theoretical model.

The theoretical curve is the Beer's law application result using the extinction coefficients we calculated.

There is a good agreement in the case of a non-scattering medium as it appears for the Christiansen peaks. However, when the fibres scatter the theoretical curve underestimates the real transmittance value.

Two reasons can explain this phenomenon:

(i) with Beer's law it is not possible to take into account the intensity due to multiple scatterings in the medium;

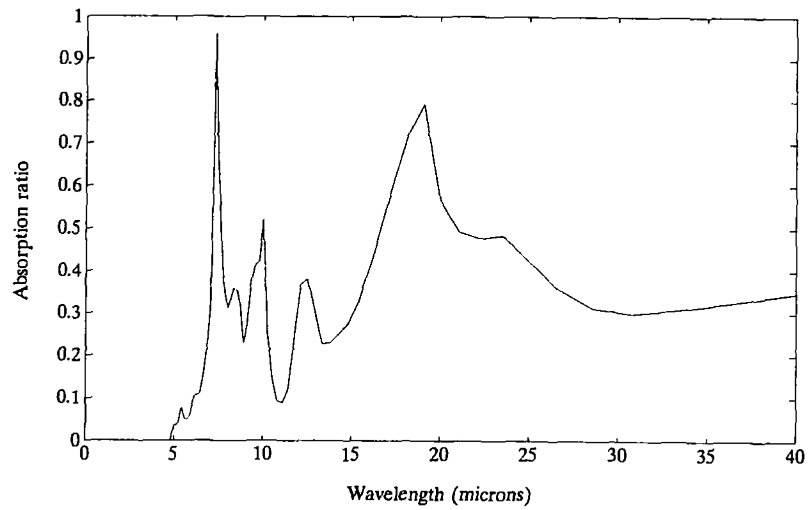

FIG. 4. Variation of the absorption ratio with the wavelength.

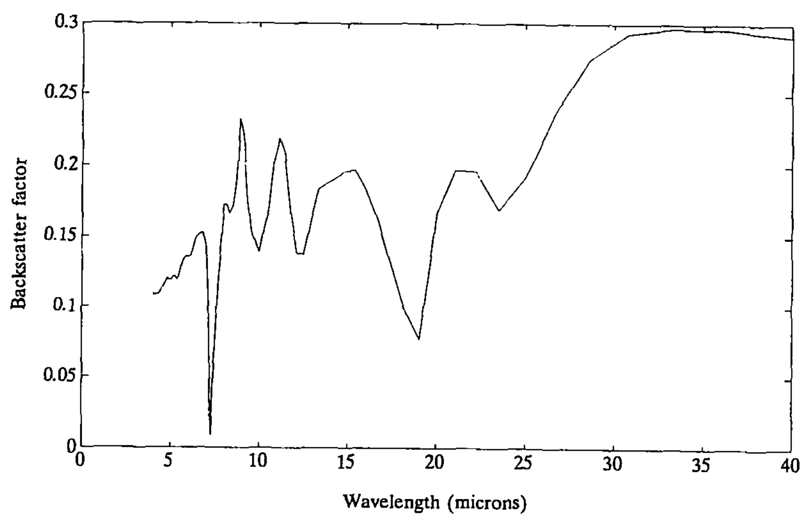

FIG. 5. Variation of the backscatter factor with the wavelength.

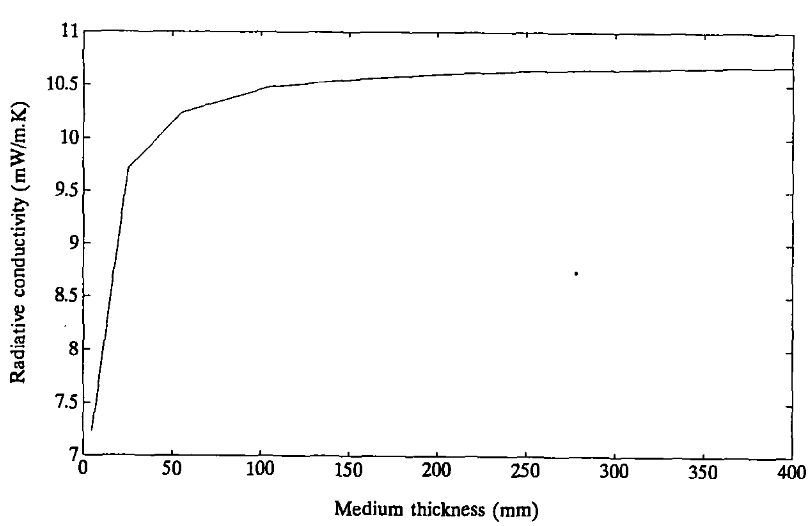

FIG. 6. Thickness effect on the radiative conductivity with a fibre diameter of 6 μm and $f_v = 0.004$.

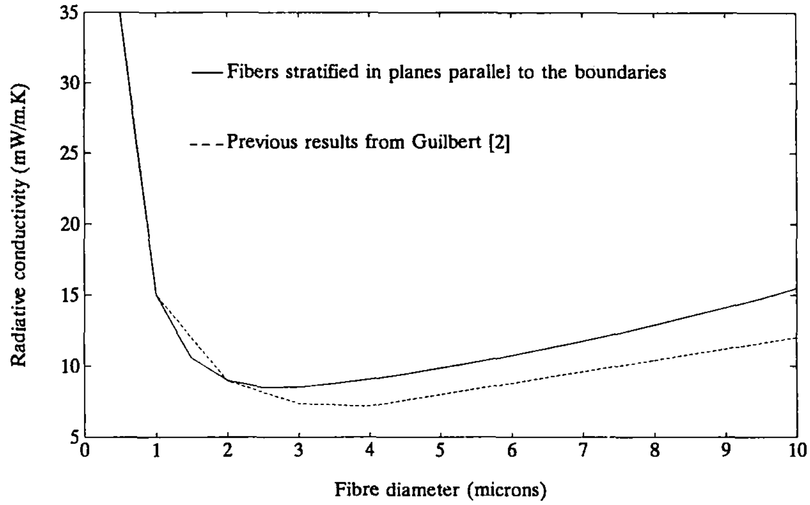

FIG. 7. Fibre diameter effect on the radiative conductivity and influence of the anisotropy of the medium. Case of one optically thick medium.

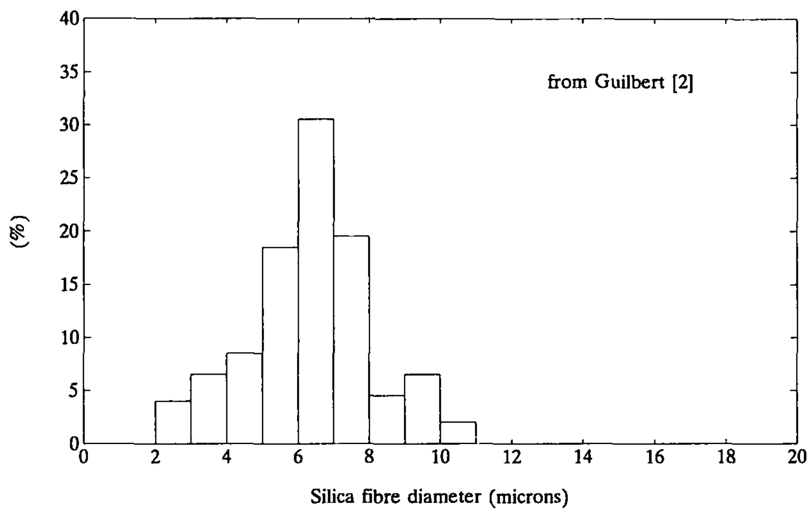

FIG. 8. Fibre diameter distribution histogram.

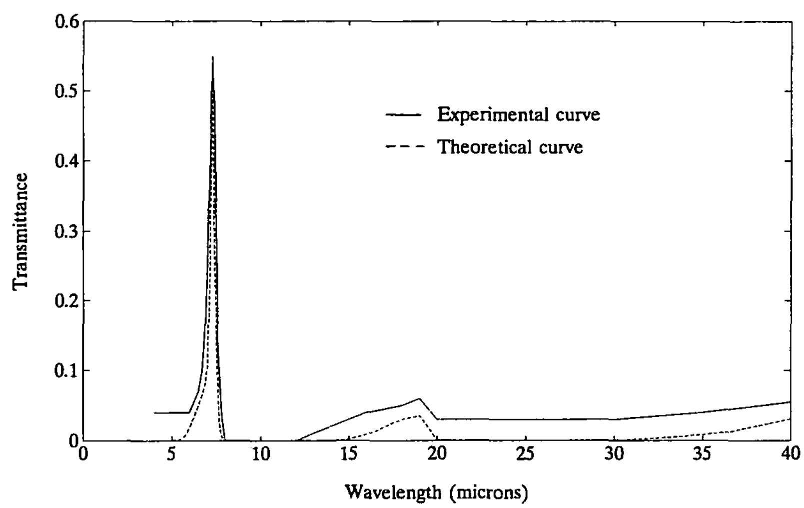

FIG. 9. Spectral transmittance for a slab with a thickness of 5 mm.

(ii) the spectrometer does not allow determination directional values since the detection is realized according to a defined solid angle.

A more accurate model including exact scattering phenomena is actually studied.

CONCLUSION

We used Lee's and Kerker's theories to calculate the fibrous medium radiative properties for the case of fibres oriented in stratification planes parallel to the boundaries. The application of our results to a layer of silica fibres allowed us to determine the absorption, scattering and extinction coefficients and the backscatter factor. In addition, we determined a radiative conductivity from a two flux model showing the influence of the medium thickness and the fibre diameter. Finally, an experimental study in transmission allowed us to confirm our theoretical results, particularly concerning the existence of the two Christiansen filters; thickness effect and the optimal fibre diameter existence being already shown by measurements of radiative conductivity.

Despite this, a complementary study remains necessary to obviate the limits assigned by the two flux model and to obtain a more reliable characterization of the pure radiative transfer.

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Correlations of pressure drop in packed beds taking into account the effect of confining wall

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INTRODUCTION

THE ABILITY to predict reliably the pressure drop through packed beds is of great significance since pumping costs are directly related to the pressure drop of the system in question. When pressure drop and flow information is required for a wide range of Reynolds number in packed bed configurations, the Ergun equation [1] has been the favourite choice amongst literature correlations. Ergun's equation effectively accounts for simultaneous inertia and viscous energy losses, and a fair amount of pressure drop-flow data have been correlated by it for beds with various geometrically shaped particles. Macdonald *et al.* [2] gave a good account of the various published models, and divided them into roughly three categories: (a) phenomenological models; (b) models based on conduit flow: (i) geometrical models, (ii) statistical models and (iii) models utilizing the complete Navier–Stokes equation; (c) models based on flow around submerged objects. There is, however, a great deal of overlap between these models. The simplest pressure drop correlation is the one proposed by Ahmed and Sunada [3]

$$-\frac{\nabla P}{\mu V_0} = \alpha + \beta \frac{\rho V_0}{\mu} \tag{1}$$

where V_0 is the superficial velocity, μ and ρ are, respectively, the viscosity and density of the fluid, and ∇P the pressure gradient. α and β are model parameters to be established empirically by a least square fitting procedure. However, despite its attractive simplicity, the most serious drawback to equation (1) is the lack of parameters characterising the porous medium. It is for this reason that Macdonald *et al.* [2] proposed a modified form of Ergun equation (2), where the constants 150 and 1.75 were replaced, respectively, by A and B

$$\frac{\Delta P}{L} = \frac{150\mu u(1-\varepsilon_m)^2}{d_p^2 \varepsilon_m^3} + \frac{1.75\rho u^2(1-\varepsilon_m)}{d_p \varepsilon_m^3} \tag{2}$$

rearranging equation (2) we obtain

$$\frac{\Delta P d_p^2 \varepsilon_m^3}{L\mu u(1-\varepsilon_m)^2} = B\frac{\rho u d_p}{\mu(1-\varepsilon_m)} + A. \tag{3}$$

In equation (3), the left-hand side is called the modified friction factor, f', while the term $\rho u d_p/\mu(1-\varepsilon_m)$ is referred to as the modified Reynolds number, Re'. Macdonald *et al.* [2] proposed a fixed value of 180 for A, while suggesting that B varied from a value of 1.8 for smooth particles to 4.0 for rough particles.

There are other Ergun based pressure drop correlations in the literature which mainly cast doubt on the universality of the constants 150 and 1.75 in the original equation (1). For example, Handley and Heggs [4] proposed a correlation having constants of 368 and 1.24, instead of the respective values of 150 and 1.75 as proposed by Ergun [1].

However, recent work on the characterisation of structure of packed beds [5, 6] has shown that the diameter ratio,